COMPARATIVE STUDY OF NUMERICAL INTEGRATION

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Abstract :

This research paper is a comparative study of the Trapezoidal rule, Simpson's 1/3 rule, and Simpson's 3/8 rule for numerical integration. It aims to evaluate and compare the accuracy and performance of these methods in approximating definite integrals. Numerical integration is essential for approximating the value of a definite integral, especially when an exact analytical solution is difficult to obtain.

Keywords : Numerical Integration, Simpson's 1/3 Rule, Simpson's 3/8 Rule, Trapezoidal Rule, Newton-Cotes Formulas, Definite Integrals.

Introduction :

The complete conception about numerical integration including Newton-Cotes formulas and comparing the rate of performance or the rate of accuracy of Trapezoidal, Simpson's 1/3 and Simpson's 3/8. To verify the accuracy, we compare each rule demonstrating the smallest error value among them. It includes graphical comparisons mentioning these methods graphically. After all, it is then methods considered, Simpson's 1/3 is more effective and accurate when the condition of the subdivision is only even for solving a definite integral. Nowadays, it is essential due to computers are too able to go through the analytic manner of integration, even associating between analytical schemes and computer processor.

The main purpose is that to evaluate the method which is the best for solving the definite function applying numerical methods.

To evaluate the performance of these tasks, a comparative analysis of the Simpson's 1/3, Simpson's 3/8 and Trapezoidal methods were discussed.

The general formula of simpson's 1/3 rule :

In numerical integration, Simpson's 1/3 rule provides a practical way to compute definite integrals. It approximates the integrand, f(x), with a parabola, effectively replacing it with a second-degree polynomial, p(x), $\int_a^b \; ydx$. The integral of this parabola is then used as an estimate of the original integral.

Now, according to Newton-Cotes general quadrature formula,

$$I = h\left[ky_0 + \frac{k^2}{2}\Delta y_0 + \left(\frac{k^3}{3} - \frac{k^2}{2}\right)\frac{\Delta^2 y_0}{2!} + \left(\frac{k^4}{4} - k^3 + k^2\right)\frac{\Delta^3 y_0}{3!} + \left(\frac{k^5}{5} - \frac{3k^4}{2} + \frac{11k^3}{3} - 3k^2\right)\frac{\Delta^4 y_0}{4!} + \ldots\right] \quad (1)$$

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This leads to the composite Simpson's 1/3 rule:

$$\int_{x_0}^{x_0+kh} ydx = \frac{h}{3}[(y_0 + y_k) + 4(y_1 + y_3 + \ldots + y_{k-1}) + 2(y_2 + y_4 + \ldots + y_{k-2})] \quad (2)$$

The general formula of simpson's 3/8 rule :

Simpson's 3/8 rule approximates definite integrals using a cubic polynomial, contrasting with the quadratic approximation of Simpson's 1/3 rule. By fitting a third-degree curve to the integrand, it provides a more accurate numerical estimate of the integral.

We apply this formula to consecutive sets of three subintervals and sum the results. This leads to the composite Simpson's 3/8 rule :

$$\int_{x_0}^{x_0+3h} ydx + \int_{x_0+3h}^{x_0+6h} ydx + \ldots + \int_{x_0+(k-3)h}^{x_0+kh} ydx$$

$$= \frac{3}{8}h\,[(y_0 + y_k) + 3(y_1 + y_2 + y_4 + y_5 + \ldots + y_{k-1}) + 2(y_3 + y_6 + \ldots + y_{k-3})] \quad (3)$$

The general formula of trapezoidal rule :

The trapezoidal rule or method is a concept in numerical analysis that approximates the definite integral by averaging the left and right sums and typically provides a better approximation than each does on its own.

The composite Trapezoidal rule :

$$\int_{x_0}^{x_0+kh} ydx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3 \ldots\ldots + y_{k-1}) + y_k] \quad (4)$$

Examples & Discussion

Problem-1: Assume that $\int_0^1 2e^{-x}dx$

is found by applying Simpson's 1/3 rule, Simpson's 3/8 rule, and the trapezoidal rule. The outcomes of the three approaches are shown in the following table, and a comparison of the approximate error is also provided below.

K	Exact Value	Simpson's 1/3 Rule	Error	Simpson's 3/8 Rule	Error	Trapezoidal Rule	Error
1	1.264241118	0.911919627	0.35232149	1.0259095	0.23833153	1.3678794	0.103638323
2	1.264241118	1.26466736	0.00042624	1.1953017	0.06893933	1.2904703	0.026229262
3	1.264241118	1.169075316	0.09516580	1.2644311	0.00019006	1.2759252	0.011684315

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4	1.264241118	1.264268351	0.00002723	1.2128637	0.05137732	1.2708188	0.00657774
5	1.264241118	1.2103129357	0.05392818	1.2420963	0.02214481	1.2684524	0.004211329
6	1.264241118	1.2642465192	0.00000540	1.2642532	0.00001211	1.2671662	0.002925129
7	1.264241118	1.226709564	0.03753155	1.2361444	0.02809667	1.2663904	0.002149338
8	1.264241118	1.2642428292	0.00000171	1.2512856	0.01295551	1.2658868	0.001645718
9	1.264241118	1.2354795060	0.02876161	1.2642435	0.00000240	1.2655415	0.001300391
10	1.264241118	1.2642418191	0.00000070	1.2449458	0.01929527	1.2652944	0.001053358
11	1.264241118	1.2409331286	0.02330798	1.2523177	0.01192337	1.2651116	0.000870569
12	1.264241118	1.2642414560	0.00000033	1.2642418	0.00000076	1.2649726	0.000731536
13	1.264241118	1.2446505081	0.01959060	1.2495557	0.01468535	1.2648644	0.000623331
14	1.264241118	1.2642413003	0.00000018	1.2759996	0.01175852	1.2647785	0.000537471
15	1.264241118	1.2473462541	0.01689486	1.2642414	0.00000031	1.2647093	0.000468202
16	1.264241118	1.2642412247	0.00000010	1.2502686	0.01397250	1.2646526	0.000411509
17	1.264241118	1.2493903661	0.01485075	1.2585101	0.00573097	1.2646056	0.000364523
18	1.264241118	1.2642411845	0.00000006	1.2642412	0.00000015	1.2645662	0.000325147
19	1.264241118	1.2509935138	0.01324760	1.2543077	0.00993333	1.2645329	0.000291823
20	1.264241118	1.2642411615	0.00000004	1.2594109	0.00483013	1.2436942	0.020546906

Here K = 1 to 20 which is the number of subdivision of the interval of the integration.

Problem-2: Assume that $\int_0^{\pi} [e^x + cos(x)]dx$

is found by applying Simpson's 1/3 rule, Simpson's 3/8 rule, and the trapezoidal rule. The outcomes of the three approaches are shown in the following table, and a comparison of the approximate error is also provided below.

K	Exact Value	Simpson's 1/3 Rule	Error	Simpson's 3/8 Rule	Error	Trapezoidal Rule	Error
1	22.14069263	25.28007421	3.13938158	28.440083	6.29939085	37.920111	15.77941868
2	22.14069263	22.71507737	0.57438474	22.720856	0.58016433	26.516335	4.375643227
3	22.14069263	18.42382232	3.71687031	22.403968	0.26327557	24.127984	1.987291473
4	22.14069263	22.184268531	0.04357590	19.721500	2.41919163	23.267285	1.126592733

5	22.14069263	18.99626913	3.1444235	21.399384	0.74130767	22.864344	0.723651526
6	22.14069263	22.14964448	0.00895185	22.160216	0.01952344	22.644229	0.503536753
7	22.14069263	19.59737903	2.5433136	20.274577	1.86611545	22.511083	0.370390827
8	22.14069263	22.14356501	0.00287238	21.465769	0.67492309	22.424495	0.283802464
9	22.14069263	20.031529046	2.10916358	22.144685	0.00399296	22.365052	0.224359462
10	22.14069263	22.14187687	0.00118424	21.115247	1.02544562	22.322493	0.181801065
11	22.14069263	20.34617915	1.79451348	21.573364	0.56732831	22.290984	0.15029159
12	22.14069263	22.14126577	0.00057314	22.141971	0.00127921	22.267006	0.12631405
13	22.14069263	20.58174186	1.55895077	20.978295	1.16239672	22.248339	0.10764669
14	22.14069263	22.14100267	0.00031004	21.659170	0.48152238	22.233522	0.09283023
15	22.14069263	19.56914089	2.57155174	22.141219	0.00052702	22.566571	0.42587932
16	22.14069263	22.14087463	0.000182	21.168853	0.97183961	22.211779	0.07108711
17	22.14069263	20.90831097	1.23238166	21.724640	0.41605178	22.203667	0.06297451
18	22.14069263	22.14080636	0.00011373	22.140947	0.00025496	22.196867	0.05617516
19	22.14069263	21.02568877	1.11500386	21.306657	0.83403528	22.191112	0.05042022
20	22.14069263	22.1407673	0.00007463	21.775270	0.36542249	22.186198	0.04550627

Here K = 1 to 20 which is the number of subdivision of the interval of the integration.

Graphical comparison of the approximate error :

Firstly, we get the following graphical comparison for problem-1

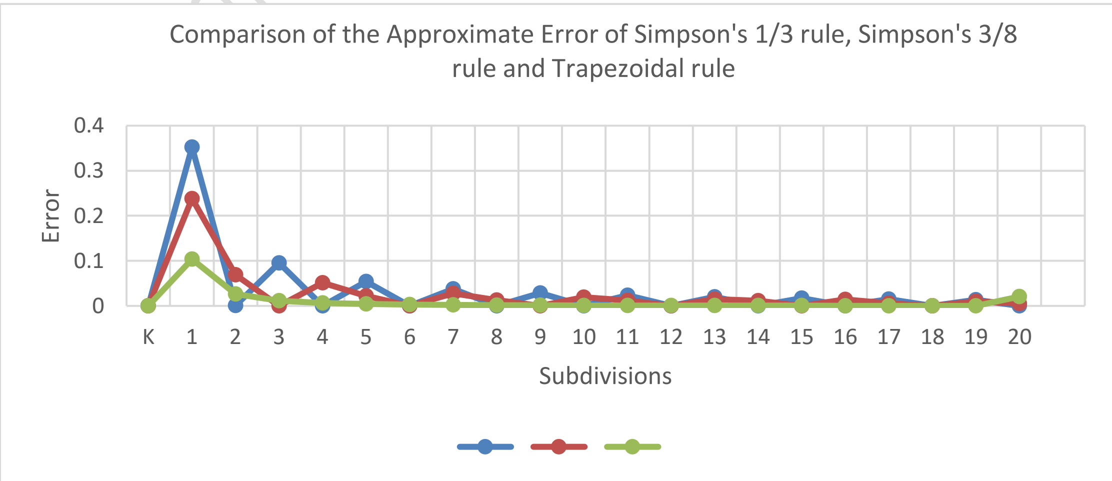

Figure 4: Comparison of the Approximate Error of Simpson's 1/3 rule,

Simpson's 3/8 rule and Trapezoidal rule

The provided graph visually demonstrates that Simpson's 1/3 rule exhibits the smallest approximate error compared to both Simpson's 3/8 rule and the Trapezoidal rule for the same integration problem. This indicates that Simpson's 1/3 rule generally provides a more accurate numerical approximation of a definite integral among these three methods.

Secondly, we obtain the following graphical comparison for problem-2

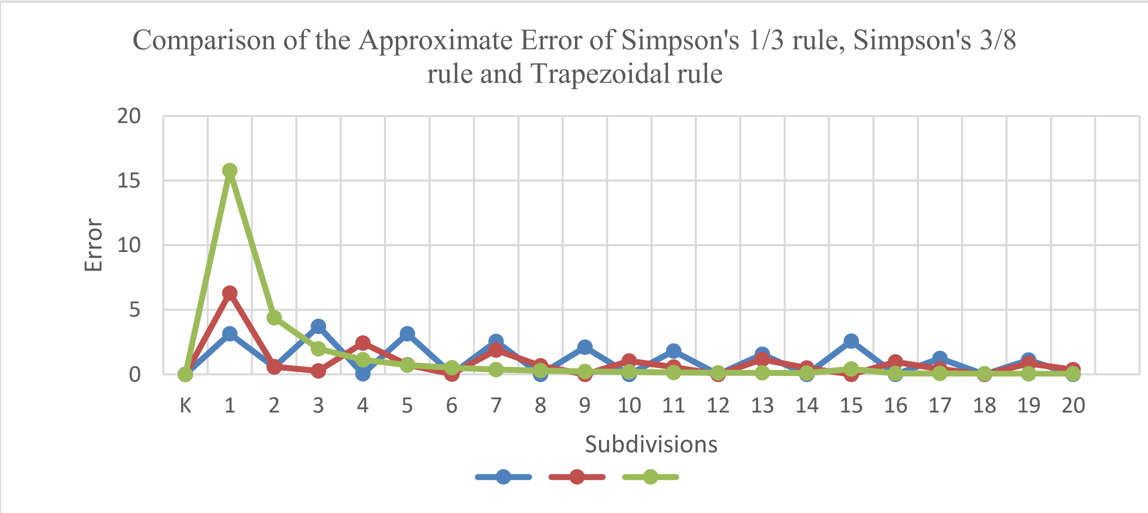

Figure 5: Comparison of the Approximate Error of Simpson's 1/3 rule,

Simpson's 3/8 rule and Trapezoidal rule

Also, the above graph visually demonstrates that Simpson's 1/3 rule exhibits the smallest approximate error compared to both Simpson's 3/8 rule and the Trapezoidal rule for the same integration problem. This indicates that Simpson's 1/3 rule generally provides a more accurate numerical approximation of a definite integral among these three methods.

Conclusion :

According to our analysis, if the function is sufficiently smooth and the interval is divided into an even number of subintervals, Simpson's 1/3 rule typically produces the smallest error when approximating definite integrals. Simpson's 1/3 rule uses quadratic polynomials to approximate the integrand, which enables a more accurate fit to the curve and results in higher accuracy than the Simpson's 3/8 and Trapezoidal rule.

References :

- Bhonsale, S. S., Telen, D., Stokbroekx, B., & Van Impe, J. (2019). Comparison of numerical solution strategies for population balance model of continuous cone mill. Powder Technology.
- Burden Richard L., Faires J.Douglas (2007): Numerical Analysis, pp. 186-213, 7th edition, Thomson Books.

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- Concepcion Ausin, M. (2007) an introduction to quadrature and other numerical integration techniques, Encyclopedia of Statistics in Quality as well as reliability. Chichester, England
- Delves, L.M. (1968): The Numerical Evaluation of Principal Value Integrals, Computer Journal, Vol. 10, , P. 389.
- Dhali, Md Nayan, M. Farhad Bulbul, and Umme Sadiya. "Comparison on Trapezoidal and Simpson’s Rule for unequal data space." International Journal of Mathematical Sciences and Computing 5.4 (2019): 33-43.
- Ohta, Koji, and Hatsuo Ishida. "Comparison among several numerical integration methods for Kramers-Kronig transformation." Applied Spectroscopy 42.6 (1988): 952-957.
- Parisi, V., & Capuzzo-Dolcetta, R. (2019). A New Method to Integrate Newtonian N-Body Dynamics. arXiv preprint arXiv:1901.02856.
- Rajesh Kumar Sinha, Rakesh Kumar, 2010, Numerical method for evaluating the integrable function on a finite interval, International Journal of Engineering Science and Technoligy. Vlo-2(6).
- Uilhoorn, Ferdinand Evert. "A comparison of numerical integration schemes for particle filter-based estimation of gas flow dynamics." Physica Scripta 93, no. 12 (2018): 125001.
- Wang, Yi, and Shuyu Sun. "Direct calculation of permeability by high-accurate finite difference and numerical integration methods." Communications in Computational Physics 20.2 (2016): 405-440.

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