# STUDY OF HYDROMAGNETIC EFFECTS ON THREE DIMENSIONAL FLOW PAST A VERTICAL POROUS PLATE

**Hariom Singh Tomar ,**
**Department of** Mathematics
Chintamani College of Arts & Science,
Gondpipari, Dist.-Chandrapur
Email-hstomar5@gmail.com

---

**Abstract:**

In this paper effect of magnetic field on the three dimensional flow of an electrically conducting viscous incompressible fluid past a porous vertical plate subjected to periodic suction velocity distribution has been analyzed. We analyses is presented for the hydro magnetic the induced magnetic field. Approximate solutions for friction have been investigated. The results are shown by the tables and graphs.

**Keywords :** Three-dimensional, Injection, Suction Parameter ,Hartmann Number , Prandtl Number

---

**Introduction :**

We consider the fully developed steady laminar flow of an incompressible ,electrically through porous medium .[1] Guria et.al.we studied of Hydromagnetic Effect on the Three Dimensional Flow Past a Vertical Porous Plate. [2]Chauhan et.al. discussed the Approximation Solution to a Three Dimensional Free Convective Flow of Hydromagnetic Porous Plate.[3] Guria et.al.Studied of Hydrodynamic effect on the three dimensional flow past a vertical porous plate. [4] Singh, discussed the Hydromagnetic effect on the three dimensional flow past a porous plate,[5]Singh,we studied Three dimensional viscous flow and heat transfer along a porous plate, [6]Gupta et.al, discussed MHD three dimensional flow past a porous plate. [7] Gersten et.al have investigated the effect of transverse sinusoidal suction velocity which leads to a three-dimensional flow over the flat surface.[8] Ramula et.al studied hydromagnetic effects on three dimensional Couette flow with transpiration cooling. The problem remained two dimensional due to the uniform injection and suction applied at the porous plates. The effect of such a suction velocity flow and heat transfer problem along flat and vertical porous plates have also been studied by [9]Singh Further[10] Singh et. al. have also analyzed the effects of transversely applied magnetic field on three dimensional Couette flow with transpiration cooling.

In this present paper we study of numerical solution to study the three dimensional flow past a vertical porous plate subjected to a periodic suction velocity distribution in the presence of a uniform magnetic field. The effects of various parameters on skin-friction are discussed with the help of a table .

**Formulation Of The Problem :**

Consider the flow of viscous, incompressible, electrically conducting fluid past along a semi -infinite vertical porous plate. The $x'$ -axis is chosen along the vertical plate that is the direction of the flow, $y'$ -axis is perpendicular to the plate and $z'$ -axis is perpendicular to the $x'$ $y'$ -plane. A uniform magnetic field B0 is imposed in the $y'$ -direction. The plate is subjected to a periodic suction velocity distribution of the form.

$$v = -v\left(1 + \in \cos \pi \frac{U_0 z}{\nu}\right) \qquad 1.$$

Denoting velocity components u`, v` , w` in the directions x ,` - y, ` - z ` -axes respectively.

The flow is governed by the following non-dimensional equations.

Continuity Equation

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (2)$$

X Component of Momentum Equation

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma B_0^2}{\rho}u \qquad (3)$$

Y Component of Momentum Equation

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1\partial p}{\rho \partial y} + \nu\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \qquad (4)$$

Z Component of Momentum Equation

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1\partial p}{\rho \partial y} + \nu\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\sigma B_0^2}{\rho}w \qquad (5)$$

Where ρ – is the density .

ν – is the kinematic viscosity.

□ – is the electrical conductivity.

$B_0$ – is the magnetic field component along $y'$-axis.

$p'$ – is the pressure

The boundary conditions are

$$y = 0, \quad u = 0, v = -v\left(1 + \in \cos \pi \frac{U_0 z}{v}\right), w = 0$$

$$y \to \infty, \quad u = U_0, v = V_0, w = 0, p = p_\infty \qquad (6)$$

We now introduce the following non-dimensional quantities

$$y' = \frac{U_0 y}{v}, \quad z' = \frac{U_0 z}{v}, \quad u' = \frac{u}{U_0}, \quad v' = \frac{v}{U_0}$$

$$w' = \frac{w}{U_0}, \quad \lambda = \frac{V}{U_0}, p = \frac{p}{\rho U_0^2}, \mathrm{M} = \frac{\sigma B_0^2 v^2}{\mu U_0^2} \qquad (7)$$

Where – Suction parameter ($\lambda$) = $V/U_0$

Hartmann Number (M) = $\frac{\sigma B_0^2 v^2}{\mu U_0^2}$

With the help of above non-dimensional variables eqn. (2) to (5) becomes.

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (8)$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - Mu \qquad (9)$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \qquad (10)$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - Mw \qquad (11)$$

The corresponding boundary condition become

$$y = 0, \quad u = 0, v = -\alpha(1 + \varepsilon \cos \pi z), w = 0$$

$$y \to \infty \quad u = 1, v = -\lambda_1, w = 0, p = p_\infty \qquad (12)$$

**Method Of Solution :**

Since the amplitude $\varepsilon \ll 1$, of the suction velocity is small, we assume the main flow velocity u (y, z) in the neighborhood of the plate as

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 U_2 + \ldots\ldots\ldots \qquad (13)$$

$$v = v_0 + \varepsilon v_1 + \varepsilon^2 V_2 + \ldots\ldots\ldots\ldots \qquad (14)$$

$$w = w_0 + \varepsilon w_1 + \varepsilon^2 W_2 + \ldots\ldots\ldots\ldots \qquad (15)$$

The similar expressions hold for other variables v, w and p. When $\varepsilon = 0$, the problem reduces to two-dimensional flow with constant injection and suction at both the plates. In this case eqn. (8) to (11) reduce to

**Zeroth -order equations :**

$$\frac{\partial v_0}{\partial y} = 0 \qquad (16)$$

$$\frac{\partial^2 u_0}{\partial y} + \lambda_1 \frac{\partial u_0}{\partial y} - M u_0 = 0 \qquad (17)$$

Where primes denote differentiation with respect to y with boundary conditions .

$$y = 0, \qquad u_0 = 0,\ v_0 = -\lambda_1$$
$$y \to \infty, \qquad u_0 = 1,\ v_0 = -\lambda_1 \qquad (18)$$

Consider the solutions of these equations are

$$u_0 = 1 - e^{r_1 y} \qquad (19)$$

With $V = -\lambda_1, w_0 = 0$ and $p_0 = p_\infty$ where $r_1 = \frac{1}{2}\left[\lambda_1 + (\lambda_1^{\,2} + 4M)^{1/2}\right]$

The coefficients of $\varepsilon$ with the help of the solution of the above two dimensional problem give the following equations.

**First order equations:**

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \qquad (20)$$

$$v_1 \frac{\partial u_0}{\partial y} + \alpha \frac{\partial v_1}{\partial z} = \left( \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) - M u_1 \qquad (21)$$

$$v_1 \frac{\partial u_0}{\partial y} - \alpha \frac{\partial u_1}{\partial y} = \left( \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) - M u_1 \tag{22}$$

$$-\alpha \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \left( \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) \tag{23}$$

$$-\alpha \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \left( \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) - M w_1 \tag{24}$$

The corresponding boundary conditions become

$$y = 0, \quad u_1 = 0, \quad v_1 = -\alpha \cos \pi z, \quad w_1 = 0$$

$$y \to \infty, \quad u_1 = 0, \quad v_1 = 0, \quad w_1 = 0, \quad p_1 = 0 \tag{25}$$

These are the partial differential equations which describe the three dimensional flow.

**Cross flow solution:**

Now we assume solutions of equations (20) to (24) of the form

$$\left.\begin{aligned} u_1(y,z) &= u_{11}(y)\cos \pi z \\ v_1(y,z) &= v_{11}(y)\cos \pi z \\ w_1(y,z) &= \frac{1}{\pi} v'_{11}(y)\sin \pi z \\ p_1(y,z) &= p_{11}(y)\cos \pi z \end{aligned}\right\} \tag{26}$$

Where the prime in $v'_{11}(y)$ denotes the differentiation with respect to y. Expression for $v_1$(y, z) and $w_1$(y, z) have been chosen so that the continuity equation (17) is satisfied. Substituting (23) in equation (18) to (21) and applying the corresponding transformed boundary conditions we get the solutions of $u_1, v_1, w_1$ and $p_1$ as

$$u_1(y,z) = \frac{\lambda_1}{r_2 - r_3}\left[c_1 e^{-r_1 y} - c_2 e^{-(r_1 - r_3)y} + c_3 e^{-(r_1 + r_2)y}\right]\cos \pi z \tag{27}$$

$$v_1(y,z) = \frac{\lambda_1}{r_2 - r_3}\left[r_3 e^{-r_2 y} - r_2 e^{-r_3 y}\right]\cos \pi z \tag{28}$$

$$w_1(y,z) = \frac{\lambda_1 r_2 r_3}{\pi (r_2 - r_3)}\left[e^{-r_2 y} - r_2 e^{-r_3 y}\right]\sin \pi z \tag{29}$$

$$p_1(y,z) = \frac{\lambda_1 r_2 r_3}{\pi(r_2 - r_3)}\left[\{r_3(r_1^* - \lambda_1) - M\}e^{-r_3 y} - \{r_2(r_1 - \lambda_1)y - M\}e^{-(r_1+r_2)}\right]\cos \pi z \tag{30}$$

Where

$$r_1 = \frac{1}{2}\left[\lambda_1 - (\lambda_1^{\,2} + 4M)^{1/2}\right]$$

$$r_2 = \frac{1}{2}\left[r_1 + (r_1^2 + 4\pi^2)^{1/2}\right]$$

$$r_3 = \frac{1}{2}\left[r_1^* + (r_1^* + 4\pi^2)^{1/2}\right]$$

$$\eta = \frac{1}{2}\left[\lambda_1 + \{\lambda_1 + 4(\pi^2 + M)\}^{1/2}\right]$$

$$C_1 = C_2 - C_3, \quad C_2 = r_2 / r_3, \quad C_3 = r_1 r_3 / r_2 (3r_1 - \lambda_1)$$

**Table 1 : Transverse velocity $w_1$ for z = – 0.5 (values in the table will take –ve sign for z = 0.5)**

| □ | M | 0 | 0.2 | 0.5 | 1.0 | 1.5 |
|---|---|---|---|---|---|---|
| 0.8 | 0 | 0 | 0.1768 | 0.1656 | 0.0647 | 0.0190 |
| | 2 | 0 | 0.1744 | 0.1627 | 0.0652 | 0.0204 |
| | 4 | 0 | 0.1720 | 0.1598 | 0.0657 | 0.0218 |
| 1.0 | 0 | 0 | 0.3723 | 0.3353 | 0.1229 | 0.0340 |
| | 2 | 0 | 0.3666 | 0.3292 | 0.1245 | 0.0371 |
| | 4 | 0 | 0.3612 | 0.3234 | 0.1259 | 0.0399 |
| 1.5 | 0 | 0 | 0.6447 | 0.6077 | 0.3953 | 0.3095 |
| | 2 | 0 | 0.6231 | 0.5853 | 0.3878 | 0.3004 |
| | 4 | 0 | 0.4103 | 0.3725 | 0.1750 | 0.0890 |

**Table 2 : Main flow skin-friction □x, at □ = 0.2, z = 0.**

| □□□ | 0 | 2 | 4 |
|---|---|---|---|
| 0.5 | 0.5112 | 1.7152 | 2.2999 |
| 1.0 | 1.0418 | 2.0673 | 2.6379 |
| 1.5 | 1.1108 | 1.7731 | 2.0282 |

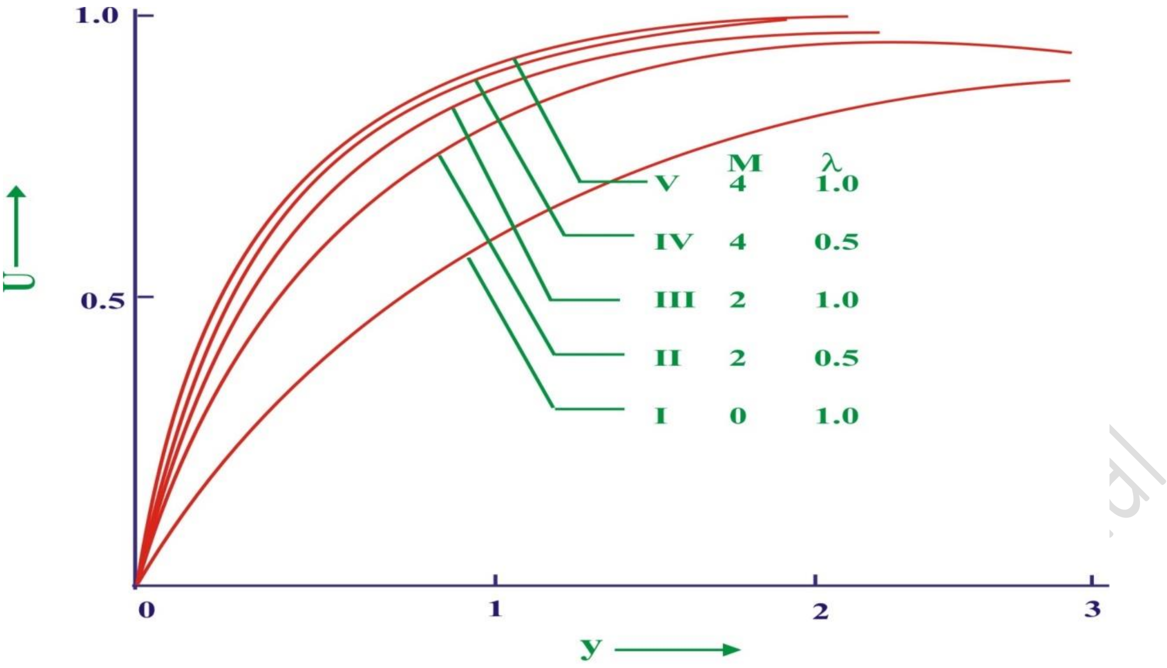


**Figure 1 : Velocity profile for ε = 0.2, z = 0.**

## [V]. RESULT:

The main flow velocity u and the transverse velocity component w, now calculated the skin friction, $\tau_z$, in the main flow direction and $\tau_x$ in the direction perpendicular to main flow, in the non-dimensional forms as

**Skin friction:**

$$\tau_x = \frac{\tau'_x}{\rho U_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u'_0(0) + \varepsilon u'_1(0)$$

On putting the value (17) and (25) after partial differential with respect to y and we have

$$\tau_x = r_1 + \frac{\varepsilon\lambda_1}{r_2 - r_3}\left[c_2(r_1 + r_3 - \eta) - c_3(r_1 + r_2 - \eta)\right]\cos \pi z \qquad (29)$$

and

$$\tau_z = \tau'_z / \rho U_0^2 = \left(\frac{\partial w}{\partial y}\right)_{y=0} = \varepsilon w'_1(0) = -\frac{\varepsilon}{\pi}\lambda_1 r_2 r_3 \sin \pi z \qquad (30)$$

Respectively .

For M = 0 the expression for $\tau_x$ and $\tau_z$ respectively reduce to

$$\tau_x = \lambda_1\left(1 + \varepsilon\frac{\eta + \lambda_1 - z}{2\eta}\cos \pi z\right) \qquad (31)$$

and

$$\tau_z = -\varepsilon\lambda_1\eta\sin\pi z \quad (32)$$

**Discussion :**

The main flow velocity profiles u are shown in figure (1) for different values of M and □ it is observed from the figure that the velocity increase of M and □ both.

The numerical values for z = – 0.5 in table 1. illustrate the trend of the transverse velocity component $w_1$. On studying of this table that shows where increase of the suction parameter, there is an increase in the transverse velocity. The skin friction in the direction normal to the main flow results from the secondary flow $w_1$ due to the symmetry at the points of maximum (z = 0) and minimum (z = 1) suction velocity, the transverse component of plate skin friction $\tau_z$ disappears at these points. However, the maximum and minimum $\tau_Z$ occurred in between these points, i.e., at z = – 0.5 and z = 0.5 respectively.

The numerical values of the main flow skin-friction are presented in table 2. It is clear from these values that $\tau_x$ increases with the increase of the suction parameters and the Hartmann number both.

**References :**


- Guria. M. , Kanch. A.K. and Jana. R.N. Hydromagnetic Effect on the Three Dimensional Flow Past a Vertical Porous Plate. Journal of Physical Sciences, Vol. 10,( 2006), 44 – 54.
- Chauhan .T. S. and I.S. Chauhan.I.S. Approximation Solution to a Three Dimensional Free Convective Flow of Hydromagnetic Porous Plate. Mathematica Aeterna, Vol. 3, (2013), no. 6, 463 - 472
- Guria.M. and Jana .R.N. Hydrodynamic effect on the three dimensional flow past a vertical porous plate," Internat. J. Math. & and Math. Sci., 20 (2005) 3359-3372.
- Singh. K. D. Hydromagnetic effect on the three dimensional flow past a porous plate. ZAMP 41 (1990), 441-446.
- Singh. K. D. Three dimensional viscous flow and heat transfer along a porous plate. Z. Angew. Math. Mech. 73 (1993), 58-61.
- Gupta. G. D. and Johari . R . MHD three dimensional flow past a porous plate, Indian J. Pure Appl. Math, Vol 32 No. 3 PP 371-386 (2001).
- Gerston .K. and Gross. J.F . The three dimensional convective flow and heat transfer through a porous medium. ZAMP, Vol. 25, (1974), pp. 399-408.
- RamulaA.S., Kishan. N. and Rao. J.A. Hydromagnetic effects on three dimensional couette flow with transpiration cooling. Bulletin of the Allahabad. Maths. Soc. Vol. 18, (2003), pp. 51-61.
- Singh.K.D. Three-dimensional Couette flow with transpiration cooling. ZAMP, Vol. 50, (1999), pp. 661-668.
- SinghK.D. MHD three-dimensional Couette flow with transpiration cooling. ZAMM. Vol. 81 (10), (2001), pp. 715-720